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LETTER TO THE EDITOR

Condon domains as sine–Gordon solitons

A Gordon† and I D Vagner‡

† Department of Physics, Oranim-School of Education of the Kibbutz Movement, Haifa University, Tivon 36910, Israel

‡ Max-Planck Institut für Festkörperforschung, Hochfeld-Magnetlabor, F-38042, 166X Grenoble Cédex, France

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Abstract. Magnetic domains and the de Haas–van Alphen effect are considered on the basis of the sine–Gordon equation being of Lagrange–Euler form for magnetic induction. The kink (soliton) solution obtained describes the domain walls of Condon domains. Calculations of the domain wall width and energy are carried out.

Shoenberg (1962) showed that the magnetisation M of real metals as a function of the magnetic field strength H can be approximated as the magnetisation of a system of electrons in the presence of the field $\boldsymbol{B} = \boldsymbol{H} + 4\pi\boldsymbol{M}$. He pointed out that if the magnetisation is large compared with ΔB, where ΔB is the period of the magnetisation oscillations, H should be replaced by B in the Lifshitz–Kosevich (1956) formula. Pippard (1963) considered the problem of magnetic interaction and showed that Shoenberg's arguments lead to a multiple-valued M as a function of the field. Under these conditions the de Haas–van Alphen effect is non-linear and hence a sample breaks into magnetic domains (Condon 1966, Condon and Walstedt 1968). The stratification of the sample into Condon domains occurs when the following condition is fulfilled: $\chi = \mathrm{d}M(B)/\mathrm{d}B > 1/4\pi$, where χ is the magnetic susceptibility. Conventional consideration of Condon domains (Privorotskii 1967, Ying and Quinn 1969, Markievicz 1986, Maniv and Vagner 1989, Gordon *et al* 1989) is based on expansions of the oscillating magnetisation in a power series in the magnetic induction. Also according to Pippard (1963) the magnetisation can be closely approximated as a sinusoidal function of the magnetic induction. Therefore, it is natural to study Condon domains on the basis of the sinusoidal behaviour of the magnetisation. In this letter we shall show that the sinusoidal dependence of the magnetisation on magnetic induction leads to new properties of Condon domains.

A detailed study of Condon domains was carried out by Privorotskii (1967). Following Pippard (1963) he suggested that the oscillating magnetisation may be represented as the following sinusoidal function of the magnetic induction:

$$4\pi M_0(B) = [(4\pi\chi - 1)/n]\sin(n\tilde{B}) \tag{1}$$

where M_0 is the magnetisation for the spatially homogeneous case, $\tilde{B} = B - (B_1 + B_2)/2$, where B_1 and B_2 are values of B in different domains, $n = 2\pi/(B_2 - B_1)$. It was shown that the correction to the magnetisation connected with the inhomogeneity is proportional to $\nabla^2 B$ in the case of weak inhomogeneity. Then the problem of minimisation of the thermodynamic potential with respect to $\tilde{B}$ for determination of the domain wall shape and width is reduced to the solution of the following Lagrange–Euler equation (Privorotskii 1967):

$$-(4\pi\chi - 1)\tilde{B} + n^2\tilde{B}^3/6 = (r_0^2/4)\,\mathrm{d}^2\tilde{B}/\mathrm{d}x^2 \tag{2}$$

where the sine in (1) is replaced by its expansion in a power series up to the term in $\tilde{B}^3$, r_0 is the cyclotron radius and the x axis is perpendicular to the transitional layer. In this case $\tilde{B}$ is of the Ginzburg–Landau form:

$$\tilde{B} = [\sqrt{6}(4\pi\chi - 1)^{1/2}/n]\tanh(x/\Delta) \tag{3}$$

where the domain wall width is given by

$$\Delta = r_0/2\sqrt{2}(4\pi\chi - 1)^{1/2}. \tag{4}$$

We do not expand the sine in (1). Then we obtain

$$-[(4\pi\chi - 1)/n]\sin(n\tilde{B}) = (r_0^2/4)\,\mathrm{d}^2\tilde{B}/\mathrm{d}x^2 \tag{5}$$

instead of (2). Equation (5) is a modification of the sine–Gordon equation (Rubinstein 1970): it is a time-independent sine–Gordon equation. The equation is integrated to give

$$\tilde{B} = (2/n)\sin^{-1}\{\mathrm{sn}[2(4\pi\chi - 1)^{1/2}x/kr_0, k]\} \tag{6}$$

where sn is the Yakobi elliptic function, k is its module $k = [2/(1 + E)]^{1/2}$ where E is the constant of the first integration (the constant of the second integration is chosen as zero: $x_0 = 0$). For $k = 1$ ($E = 1$) we have

$$\tilde{B} = (2/n)\sin^{-1}\{\tanh[2(4\pi\chi - 1)^{1/2}x/r_0]\}. \tag{7}$$

The solution (7) corresponds to boundary conditions of the domain type

$$\tilde{B}(+\infty) = \pi/n \qquad \tilde{B}(-\infty) = -\pi/n \qquad \mathrm{d}\tilde{B}(\pm\infty)/\mathrm{d}x = 0. \tag{8}$$

Equation (7) is equivalent to

$$\tilde{B} = (4/n)\tan^{-1}\{\exp[2(4\pi\chi - 1)^{1/2}x/r_0]\} - \pi/n. \tag{9}$$

The solution (7) is a kink giving a domain wall: for $x \to +\infty$, $\tilde{B} = (B_2 - B_1)/2$, corresponding to the value of the magnetic induction in one domain $B = B_2$; for $x = \to -\infty$, $\tilde{B} = -(B_2 - B_1)/2$, corresponding to the value of the magnetic induction in another domain $B = B_1$. The width of the domain (7), Δ, is different from (3) and is equal to

$$\Delta = r_0/2(4\pi\chi - 1)^{1/2}. \tag{10}$$

We suppose here that the width of the transition layer is small compared with the width of domains themselves. This justifies the boundary conditions (8). For this reason the domain wall may be regarded as planar and the problem as one-dimensional. Calculations of the surface tension of the domain wall (7) give the following expression:

$$\sigma = r_0(4\pi\chi - 1)^{1/2}(B_2 - B_1)^2/\pi^2. \tag{11}$$

Equation (11) is different from the expression obtained by Privorotskii (1967) for the domain wall energy:

$$\sigma = r_0(4\pi\chi - 1)^{1/2}(B_2 - B_1)^2/24\sqrt{2}\pi. \tag{12}$$

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